

Gravitational Memory Charges of Supertranslation and Superrotation on Rindler Horizons

Masahiro Hotta, Jose Trevison, and Koji Yamaguchi
*Graduate School of Science, Tohoku University,
 Sendai 980-8578, Japan*

Abstract

In a Rindler-type coordinate system spanned in a region outside of a black hole horizon, we have nonvanishing classical holographic charges as soft hairs on the horizon for stationary black holes. Taking a large black hole mass limit, the spacetimes with the charges are described by asymptotic Rindler metrics. We construct a general theory of gravitational holographic charges for a (1+3)-dimensional linearized gravity field in the Minkowski background with Rindler horizons. Although matter crossing a Rindler horizon causes horizon deformation and a time-dependent coordinate shift, that is, gravitational memory, the supertranslation and superrotation charges on the horizon can be defined during and after its passage through the horizon. It is generally proven that holographic states on the horizon cannot store any information about absorbed perturbative gravitational waves. However, matter crossing the horizon really excites holographic states. By using gravitational memory operators, which consist of the holographic charge operators, we suggest a resolution of the no-cloning paradox of quantum information between matter falling into the horizon and holographic charges on the horizon from the viewpoint of the contextuality of quantum measurement.

1 Introduction

Recently Hawking, Perry, and Strominger (HPS) [1] [2] proposed an interesting scenario that may resolve the information loss problem [3]. They suggest that quantum information about collapsing matter is stored in an infinite number of conserved Noether currents having asymptotic symmetries, including supertranslation on a horizon. This is expected to maintain the unitarity of quantum gravity, and all of the information may be accessible in the region outside of the horizon. By incorporating superrotation symmetry on a horizon, this symmetry-based scenario may also provide a possibility of revealing the statistical mechanical origin of the Bekenstein–Hawking entropy $\mathcal{A}/(4G)$, as already pointed out in previous papers [4] and [5] by one of the authors of this paper. The nonvanishing holographic charges of these asymptotic symmetries can yield a huge number of different physical states with the same ADM energy and angular momentum. The degeneracy is so large that it may account for the order of $\mathcal{A}/(4G)$ [5].

In holographic charge arguments, the physical degrees of freedom emerge from would-be gauge degrees of freedom of the general covariance. This can be grasped easily by recalling the Poincaré covariance as a simple example. In fact, the Lorentz transformation is a subgroup of the Poincaré group and generates an infinite number of physical states with different values of the momentum. The Poincaré transformation is a subgroup of general coordinate transformations. Because general relativistic theories have the general covariance as the gauge symmetry, the Poincaré transformation can be regarded as a would-be gauge transformation, which actually causes a transition between physical states. On a horizon, a similar mechanism works, and an infinite-dimensional asymptotic symmetry appears.

In the spirit of Brown and Henneaux [6], Strominger first suggested in [7] that a three-dimensional black hole entropy is derived using a Virasoro algebra as an asymptotic symmetry at spatial infinity. Carlip [8] proposed an asymptotic Virasoro symmetry on a black hole horizon and argued that the black hole entropy is derived using the Cardy formula with a macroscopically large central charge. However, the original argument of Carlip encountered various types of criticism [9][10][11][12][13] and remains controversial. The existence of supertranslation and superrotation as a consistent asymptotic symmetry on a stationary horizon of a four-dimensional Schwarzschild black hole was first reported in a paper [4] by one of the authors of the present

paper. Subsequently, it was explicitly demonstrated that falling matter really excites charged states of the asymptotic symmetry in a three-dimensional black hole spacetime [5]. It is not known yet whether a gravitational wave excites charged states of supertranslation and superrotation on a horizon. HPS recently revisited supertranslation and superrotation on the horizon [1] [2] by using a different coordinate system from that in [4] and [5].

Using a coordinate system, HPS introduced asymptotic metrics near a horizon as

$$ds^2 = 2dvdr + g_{AB}dx^A dx^B + O(r - r_H), \quad (1)$$

where the horizon is located at $r = r_H$ and uppercase Roman letters run over spatial coordinates on the horizon. Based on the above form, HPS argue that stationary black holes do not carry classical supertranslation hair because the holographic charge vanishes in the classical level [2]. It is worth noting that the coordinate system in eq. (1) covers both regions inside of the horizon and outside of the horizon, and can be physically implemented by free-falling block-numbered clocks distributed in the space near the horizon, as depicted in the left panel of figure 1. Because of the clock free motion, it is very natural from the viewpoint of equivalence principle that nonzero holographic charges on the horizon cannot be observed. Apart from HPS's setup, if we adopt a Rindler-type coordinate system in which an asymptotic near-horizon metric is given by

$$ds^2 = 2 \exp\left(-\frac{\rho}{\kappa}\right) dv' d\rho + g_{AB}dx^A dx^B + O\left(\exp\left(-\frac{2\rho}{\kappa}\right)\right), \quad (2)$$

where κ is Rindler acceleration and the horizon is located at $\rho = \infty$, we indeed have nonvanishing classical holographic charges on the horizon even for stationary black holes [4]. The coordinate system in eq. (2) is implemented by accelerating block-numbered clocks distributed in the space near the horizon, as depicted in the right panel of figure 1. The appearance of the charges $Q[\xi]$ in the accelerated coordinate system is reminiscent of that of a thermal bath in the Unruh effect [14].

As in the black hole complementarity scenario [15], the HPS scenario requires some mechanism to avoid the no-cloning paradox. If all of the information about collapsing matter is stored in the conserved charges on the horizon, we may make a precise copy of the quantum information of the matter inside of the horizon by using the charge information. Although this

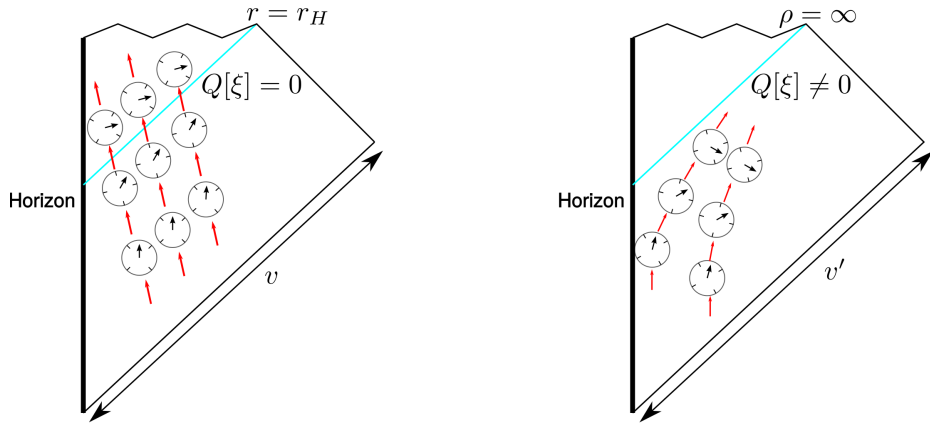


Figure 1: (Left) The near-horizon coordinate system of HPS is physically implemented by free-falling block-numbered clocks distributed in the space. In this coordinate system, classical holographic charges $Q[\xi]$ on the horizon vanish for stationary black holes. (Right) A coordinate system implemented by accelerating block-numbered clocks distributed in the space near the horizon. In this coordinate system, non-zero classical holographic charges $Q[\xi]$ appear on the horizon as soft black hole hair even for stationary black holes. The appearance of the charges in the accelerated coordinate system is reminiscent of that of a thermal bath in the Unruh effect.

naively seems to contradict the no-cloning theorem of quantum mechanics [16], HPS have not yet provided any plausible resolution of this paradox. Let us consider the gravitational collapse depicted in the left panel of figure 2. In this scenario, information about the collapsing matter is imprinted in the holographic charge states of asymptotic symmetries on the horizon. If we throw additional matter into the black hole, as depicted in the right panel of figure 2, a new horizon appears and encloses the old horizon. Then new holographic charge states have to carry all of the information about the original collapsing matter and additional matter. Naively, this appears strange. The new holographic charges must remember quantum information about the behavior of the original collapsing matter before the additional matter arrived. Thus, we potentially have duplicate quantum information about the collapsing matter on the two different horizons. This challenges the no-cloning theorem again. This situation remains unchanged even if we take a large mass limit on the black hole. In this limit, the near-horizon geometry is merely a Minkowski spacetime, and each horizon coincides with one of the Rindler horizons. The situation is depicted in figure 3. Thus, even if a linearized theory of quantum gravity is considered in a Minkowski background with Rindler horizons, the no-cloning problem should be resolved properly. This implies that the investigation of holographic charges on Rindler horizons is valuable.

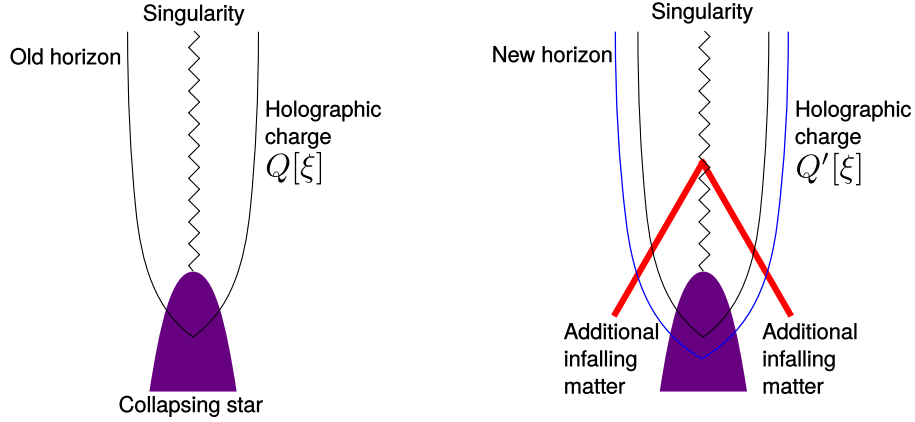


Figure 2: (Left) Collapsing matter information is imprinted in holographic charge states of asymptotic symmetries on the horizon. (Right) After throwing additional matter, the new holographic charge states have to carry whole information: original + additional.

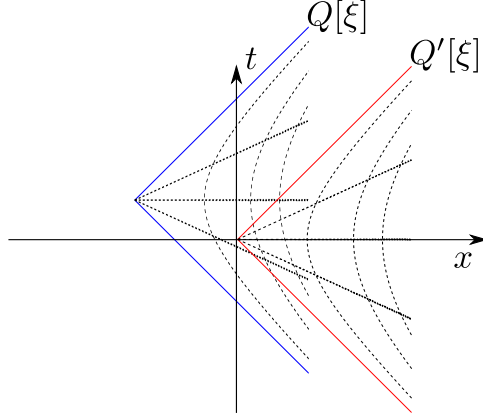


Figure 3: In the large mass limit of the black hole where each horizon coincides with one of the Rindler horizons, we potentially have duplicated quantum information.

To analyze the above problems of asymptotic Rindler spacetimes, we construct a general theory of gravitational holographic charges for a (1+3)-dimensional linearized gravity field in section 2 of this paper. Although matter crossing a Rindler horizon generates horizon deformation and a time-dependent coordinate shift, that is, gravitational memory, the charges of supertranslation and superrotation on the horizon can be defined during and after its passage through the horizon. In particular, it is verified that the charges become time-independent after matter absorption by the horizon. It should be emphasized that the time independence of the charge is very nontrivial for a general coordinate transformation. For instance, a Noether current for an infinitesimal coordinate transformation $\delta_\epsilon x^\mu = \epsilon^\mu(x)$ is given in Einstein gravitational theory by

$$J^\mu = \partial_\nu \left(\frac{\sqrt{-g}}{16\pi G} (\nabla^\mu \epsilon^\nu - \nabla^\nu \epsilon^\mu) \right)$$

and is locally conserved: $\partial_\mu J^\mu = 0$. The charge Q_ϵ is defined by the integration of J^0 in a bulk region Σ as

$$Q_\epsilon = \int_\Sigma J^0 d^3x.$$

Even though $\partial_\mu J^\mu = 0$ holds, the time independence of Q_ϵ is not generally ensured. This is because the surface integral of the flux \vec{J} does not disappear

for many $\epsilon^\mu(x)$ even if we have no gravitational wave and matter on the surface. From the local balance relation of conserved currents,

$$\frac{dQ_\epsilon}{dt} = - \int_{\partial\Sigma} \vec{J} \cdot d\vec{S},$$

Q_ϵ can vary in time because the right-hand side of the above equation is generally nonzero. Thus, for a surface of interest such as a horizon, the time independence of the holographic charges has to be checked independently by confirming that $\partial_\mu J^\mu = 0$. In section 2, we show the time independence of the holographic charges on Rindler horizons. A general formula for conserved holographic charges after the horizon absorbs the matter is provided. It is also proven that holographic states on the horizon cannot store any information about absorbed perturbative gravitational waves. To show the memory effect of horizons, a measure of the classical gravitational memory $M(x_h^+, x_h^-)$ on a future horizon at $x^- = x_h^-$ with a Rindler wedge located at $(x^+, x^-) = (x_h^+, x_h^-)$ is introduced by time integration of the holographic charges. It differs from the standard gravitational memory that appears in [17] [18] [19] [20] [21] and provides a more profound insight because it consists of conserved charges.

In section 3, we introduce a quantum gravitational memory operator $\hat{M}(x_h^+, x_h^-)$ for a Rindler horizon and propose a possible resolution of the no-cloning paradox from the viewpoint of quantum measurement contextuality. The energy-momentum tensor of quantum matter inside of a horizon does not commute with $\hat{M}(x_h^+, x_h^-)$ on the horizon. This also leads to non-commutativity between holographic charges of different horizons. Thus, a measurement of $\hat{M}(x_h^+, x_h^-)$ on a horizon affects other $\hat{M}(x_h'^+, x_h'^-)$ and quantum states of matter inside of the horizon. They are not independent observables, and the reality of $\hat{M}(x_h^+, x_h^-)$ is subject to the contextuality of quantum measurement. On the basis of this fact, we propose a conjecture that the holographic charge reality is conditioned to measurements by appropriate physical detectors. Then no cloning paradox arises, at least in the first order of perturbative quantum gravity. In a method similar to that of Unruh–DeWitt particle detectors [22] [23] for Unruh radiation in a Rindler spacetime, physical detectors accumulating information for evaluation of a quantum metric near the horizon may observe the holographic charges as a reality. Owing to gravitational interaction between infalling matter and the metric detectors distributed in the space, the detectors share quantum entanglement with the matter inside of the horizon. If we place no detector

to measure the quantum metric near the horizon, the absorbed matter is not decohered by the measurement. All of the quantum information about the infalling matter remains carried by the matter itself. Hence, the holographic Noether charges are not physical objects that share quantum entanglement with the matter inside but merely a gauge freedom of general covariance in this case. Therefore, even in quantum theory, the horizon cannot be regarded as a real holographic screen spanning the space in the absence of measurement devices. This conjecture avoids the no-cloning paradox and also supports a conservative conjecture for the information loss problem in [24], [25] and [26], which is quite different from that in the firewall conjecture [27]. In this paper, the natural units are adopted: $c = \hbar = 1$.

2 Holographic Charge Shift by Infalling Matter and Gravitational Wave

In this section, we construct a general theory of gravitational holographic charges for a weak gravity field. First, a useful gauge condition, referred to as the Rindler gauge, is introduced for an arbitrary configuration of the field. Next, using the Rindler gauge, a Regge–Teitelboim canonical theory is formulated for holographic charges on a Rindler horizon.

2.1 Weak Gravity Field and Rindler Gauge Fixing

Using the gravitational constant G , let us introduce the Planck length constant $\kappa = \sqrt{16\pi G}$. Then a perturbative gravitational field is written as

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \quad (3)$$

where $\eta_{\mu\nu}$ is the Minkowski metric, and $h_{\mu\nu}$ is a weak field in an arbitrary gauge. To consider a future Rindler horizon $x = t$, let us introduce light-cone coordinates $x^\pm = x \pm t$ for convenience. Consider the conformal Rindler coordinates (τ, ρ) ,

$$x^+ = 2\kappa e^{-\frac{\rho-\tau}{2\kappa}},$$

$$x^- = 2\kappa e^{-\frac{\rho+\tau}{2\kappa}}$$

in the (x^+, x^-) plane. In the four-dimensional coordinates $(\sigma^\mu) = (\tau, \rho, y, z)$, the background metric $\bar{g}_{\mu\nu}$ takes the standard Rindler form as

$$\begin{bmatrix} \bar{g}_{\tau\tau} & \bar{g}_{\tau\rho} & \bar{g}_{\tau y} & \bar{g}_{\tau z} \\ \bar{g}_{\rho\tau} & \bar{g}_{\rho\rho} & \bar{g}_{\rho y} & \bar{g}_{\rho z} \\ \bar{g}_{y\tau} & \bar{g}_{y\rho} & \bar{g}_{yy} & \bar{g}_{yz} \\ \bar{g}_{z\tau} & \bar{g}_{z\rho} & \bar{g}_{zy} & \bar{g}_{zz} \end{bmatrix} = \begin{bmatrix} -\Delta & 0 & 0 & 0 \\ 0 & \Delta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (4)$$

where

$$\Delta = \exp\left(-\frac{\rho}{\kappa}\right) = \frac{x^+ x^-}{4\kappa^2}.$$

Under a coordinate transformation, the weak field $h_{\mu\nu}$ in eq. (3) is transformed as

$$\varphi_{\mu\nu} = \frac{\partial x^\alpha}{\partial \sigma^\mu} \frac{\partial x^\beta}{\partial \sigma^\nu} h_{\alpha\beta},$$

and the total metric is given by

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa \varphi_{\mu\nu}.$$

Under an infinitesimal general coordinate transformation $\delta_\theta \sigma^\mu = \kappa \Theta^\mu(\sigma)$ for the metric, $\varphi_{\mu\nu}$ changes as follows:

$$\varphi_{\mu\nu}^{(R)} = \varphi_{\mu\nu} + \nabla_\mu \Theta_\nu + \nabla_\nu \Theta_\mu.$$

Here $\varphi_{\mu\nu}^{(R)}$ stands for metric perturbation in a new gauge in the (τ, ρ, y, z) coordinates. By using this gauge freedom of Θ_μ , we can always impose on $\varphi_{\mu\nu}^{(R)}$ the gauge condition

$$\varphi_{\rho\mu}^{(R)} = 0, \quad (5)$$

which is referred to as the Rindler gauge in this paper. In the flat coordinates (x^+, x^-, y, z) , the Rindler gauge imposes

$$\varphi_{\rho\rho}^{(R)} = \frac{1}{4\kappa^2} \left[(x^+)^2 h_{++}^{(R)} + 2x^+ x^- h_{+-}^{(R)} + (x^-)^2 h_{--}^{(R)} \right] = 0, \quad (6)$$

$$\varphi_{\tau\rho}^{(R)} = -\frac{1}{4\kappa^2} \left[(x^+)^2 h_{++}^{(R)} - (x^-)^2 h_{--}^{(R)} \right] = 0, \quad (7)$$

$$\varphi_{\rho A}^{(R)} = -\frac{1}{2\kappa} \left(x^+ h_{+A}^{(R)} + x^- h_{-A}^{(R)} \right) = 0, \quad (8)$$

where $h_{\mu\nu}^{(R)}$ is defined as

$$h_{\mu\nu}^{(R)} = \frac{\partial\sigma^\alpha}{\partial x^\mu} \frac{\partial\sigma^\beta}{\partial x^\nu} \varphi_{\alpha\beta}^{(R)}.$$

The $\tau\tau$ component given by

$$\varphi_{\tau\tau}^{(R)} = \frac{1}{4\kappa^2} \left[(x^+)^2 h_{++}^{(R)} - 2x^+ x^- h_{+-}^{(R)} + (x^-)^2 h_{--}^{(R)} \right]$$

is rewritten as

$$\varphi_{\tau\tau}^{(R)} = \frac{(x^-)^2}{\kappa^2} h_{--}^{(R)} = O\left((x^-)^2\right)$$

using eqs. (6) and (7). Similarly, the τA component defined as

$$\varphi_{\tau A}^{(R)} = \frac{1}{2\kappa} \left(x^+ h_{+A}^{(R)} - x^- h_{-A}^{(R)} \right)$$

is computed as

$$\varphi_{\tau A}^{(R)} = -\frac{x^-}{\kappa} h_{-A}^{(R)} = O\left(x^-\right)$$

from eq. (8). These equations can be summarized as the following asymptotic condition around $x^- = 0$:

$$\begin{bmatrix} \varphi_{\tau\tau}^{(R)} & \varphi_{\tau\rho}^{(R)} & \varphi_{\tau y}^{(R)} & \varphi_{\tau z}^{(R)} \\ \varphi_{\rho\tau}^{(R)} & \varphi_{\rho\rho}^{(R)} & \varphi_{\rho y}^{(R)} & \varphi_{\rho z}^{(R)} \\ \varphi_{y\tau}^{(R)} & \varphi_{y\rho}^{(R)} & \varphi_{yy}^{(R)} & \varphi_{yz}^{(R)} \\ \varphi_{z\tau}^{(R)} & \varphi_{z\rho}^{(R)} & \varphi_{zy}^{(R)} & \varphi_{zz}^{(R)} \end{bmatrix} = \begin{bmatrix} O\left((x^-)^2\right) & 0 & O\left(x^-\right) & O\left(x^-\right) \\ 0 & 0 & 0 & 0 \\ O\left(x^-\right) & 0 & O\left((x^-)^0\right) & O\left((x^-)^0\right) \\ O\left(x^-\right) & 0 & O\left((x^-)^0\right) & O\left((x^-)^0\right) \end{bmatrix}. \quad (9)$$

Thus, in the Rindler gauge, any weak field takes the form of eq. (9) around the Rindler horizon of $x^- = 0$. In the flat coordinates (x^+, x^-, y, z) , the transformation to the Rindler gauge is expressed as

$$h'_{\mu\nu} = h_{\mu\nu} + \partial_\mu \theta_\nu + \partial_\nu \theta_\mu,$$

where

$$\theta_\mu(x) = \frac{\partial\sigma^\alpha}{\partial x^\mu} \Theta_\alpha(\sigma).$$

For $\theta_\mu(x)$, the $\rho\rho$ component gauge condition $\varphi_{\rho\rho}^{(R)} = 0$ becomes

$$\begin{aligned} & (x^+)^2 \partial_+ \theta_+ + x^+ x^- (\partial_+ \theta_- + \partial_- \theta_+) + (x^-)^2 \partial_- \theta_- \\ &= -\frac{1}{2} \left[(x^+)^2 h_{++} + 2x^+ x^- h_{+-} + (x^-)^2 h_{--} \right]. \end{aligned} \quad (10)$$

The second gauge condition, $\varphi_{\rho r}^{(R)} = 0$, yields

$$(x^+)^2 \partial_+ \theta_+ - (x^-)^2 \partial_- \theta_- = -\frac{1}{2} \left[(x^+)^2 h_{++} - (x^-)^2 h_{--} \right], \quad (11)$$

and the third gauge condition, $\varphi_{\rho A}^{(R)} = 0$, gives

$$x^+ (\partial_+ \theta_A + \partial_A \theta_+) + x^- (\partial_- \theta_A + \partial_A \theta_-) = - (x^+ h_{+A} + x^- h_{-A}). \quad (12)$$

The above equations determine the gauge parameters $\theta_\mu(x)$ for given $h_{\mu\nu}$. Next, let us solve the equations explicitly using a variable s defined as

$$s = \frac{x^-}{x^+}.$$

Eq. (10) reads

$$\begin{aligned} & \partial_+|_s (\theta_+ + s\theta_-) \\ &= -\frac{1}{2} \left[h_{++} (x^+, sx^+, y, z) + 2sh_{+-} (x^+, sx^+, y, z) + s^2 h_{--} (x^+, sx^+, y, z) \right], \end{aligned}$$

where $\partial_+|_s$ is the partial derivative with respect to x^+ for a fixed s . Integration with respect to x^+ gives the following equation.

$$\begin{aligned} \theta_+ &= -s\theta_- + \Lambda'(s, y, z) \\ &\quad - \frac{1}{2} \int_0^{x^+} \left[h_{++} (q, sq, y, z) + 2sh_{+-} (q, sq, y, z) + s^2 h_{--} (q, sq, y, z) \right] dq, \end{aligned} \quad (13)$$

where $\Lambda'(s, y, z)$ is an unfixed integration function of s, y, z . Subtracting eq. (11) from eq. (10) gives

$$\begin{aligned}
& (x^+)^2 \partial_+ \theta_+ + x^+ x^- (\partial_+ \theta_- + \partial_- \theta_+) + (x^-)^2 \partial_- \theta_- \\
& - \left[(x^+)^2 \partial_+ \theta_+ - (x^-)^2 \partial_- \theta_- \right] \\
& = -\frac{1}{2} \left[(x^+)^2 h_{++} + 2x^+ x^- h_{+-} + (x^-)^2 h_{--} \right] \\
& + \frac{1}{2} \left[(x^+)^2 h_{++} - (x^-)^2 h_{--} \right].
\end{aligned}$$

Using the (x^+, s) variables, this is expressed as

$$\left(\partial_+ |_s + \frac{s}{x^+} \partial_s |_+ \right) \theta_- + \frac{1}{x^+} \partial_s |_+ \theta_+ = -(h_{+-} + s h_{--}).$$

Substituting eq. (13) into the above equation yields

$$\begin{aligned}
& \left(\partial_+ |_s - \frac{1}{x^+} \right) \theta_- \\
& = -(h_{+-} + s h_{--}) - \frac{1}{x^+} \partial_s |_+ \Lambda'(s, y, z) \\
& + \frac{1}{2x^+} \int_0^{x^+} q \left[\partial_- h_{++}(q, sq, y, z) + 2s \partial_- h_{+-}(q, sq, y, z) + s^2 \partial_- h_{--}(q, sq, y, z) \right] dq \\
& + \frac{1}{x^+} \int_0^{x^+} [h_{+-}(q, sq, y, z) + s h_{--}(q, sq, y, z)] dq.
\end{aligned}$$

By integrating the equation with respect to x^+ , we have a general solution of θ_- such that

$$\begin{aligned}
\theta_- & = \partial_s \Lambda'(s, y, z) - \frac{x^+}{\kappa} \frac{2}{1+s} T'(s, y, z) \\
& - \int_0^{x^+} [h_{+-}(q, sq, y, z) + s h_{--}(q, sq, y, z)] dq \\
& - \frac{1}{2} \int_0^{x^+} q \left[\partial_- h_{++}(q, sq, y, z) + 2s \partial_- h_{+-}(q, sq, y, z) + s^2 \partial_- h_{--}(q, sq, y, z) \right] dq \\
& + \frac{x^+}{2} \int_0^{x^+} [\partial_- h_{++}(q, sq, y, z) + 2s \partial_- h_{+-}(q, sq, y, z) + s^2 \partial_- h_{--}(q, sq, y, z)] dq,
\end{aligned} \tag{14}$$

where $T'(s, y, z)$ is an unfixed integration function. From this result and eq. (13), θ_+ is solved as

$$\begin{aligned}
\theta_+ &= \Lambda'(s, y, z) - s\partial_s|_+ \Lambda'(s, y, z) + \frac{x^+}{\kappa} \frac{2s}{1+s} T'(s, y, z) \\
&- \frac{1}{2} \int_0^{x^+} [h_{++}(q, sq, y, z) + 2sh_{+-}(q, sq, y, z) + s^2 h_{--}(q, sq, y, z)] dq \\
&+ s \int_0^{x^+} [h_{+-}(q, sq, y, z) + sh_{--}(q, sq, y, z)] dq \\
&+ \frac{s}{2} \int_0^{x^+} q [\partial_- h_{++}(q, sq, y, z) + 2s\partial_- h_{+-}(q, sq, y, z) + s^2 \partial_- h_{--}(q, sq, y, z)] dq \\
&- \frac{x^+ s}{2} \int_0^{x^+} [\partial_- h_{++}(q, sq, y, z) + 2s\partial_- h_{+-}(q, sq, y, z) + s^2 \partial_- h_{--}(q, sq, y, z)] dq.
\end{aligned} \tag{15}$$

Similarly, eq. (12) is rewritten as

$$\partial_+|_s \theta_A = -(h_{+A} + sh_{-A}) - \partial_A (\theta_+ + s\theta_-),$$

and the gauge parameter θ_A can be solved as follows.

$$\begin{aligned}
\theta_A &= R'_A(s, y, z) - x^+ \partial_A \Lambda'(s, y, z) \\
&- \int_0^{x^+} [h_{+A}(q, sq, y, z) + sh_{-A}(q, sq, y, z)] dq \\
&- \frac{1}{2} \partial_A \int_0^{x^+} q [h_{++}(q, sq, y, z) + 2sh_{+-}(q, sq, y, z) + s^2 h_{--}(q, sq, y, z)] dq \\
&+ \frac{x^+}{2} \partial_A \int_0^{x^+} [h_{++}(q, sq, y, z) + 2sh_{+-}(q, sq, y, z) + s^2 h_{--}(q, sq, y, z)] dq,
\end{aligned} \tag{16}$$

where $R'_A(s, y, z)$ is an unfixed integration function.

Next, let us focus on a case in which an incoming matter field or gravitational wave takes nonzero values in a region $[x_i^+, x_f^+]$ with $x_i^+ > 0$ and

vanishes outside of it. At the initial time $x^+ = 0$, the gravity field is in the vacuum state. Thus, we can set a boundary condition as

$$T' = \Lambda' = R'_A = 0.$$

The matter crossing the horizon at $x^- = 0$ induces nonzero values of $T(0, y, z)$, $R_A(0, y, z)$, $\Lambda(0, y, z)$, $\partial_s \Lambda(0, y, z)$ after the pass of the matter ($x^+ > x_f^+$) such that

$$\begin{aligned}\theta_- &= -2\frac{x^+}{\kappa}T(0, y, z) + \partial_s \Lambda(0, y, z), \\ \theta_+ &= \Lambda(0, y, z), \\ \theta_A &= R_A(0, y, z) - x^+ \partial_A \Lambda(0, y, z).\end{aligned}\tag{17}$$

Note that the above coordinate transformations yield a time-dependent metric even after matter passes across the horizon. Because eq. (17) implies that $\theta^- = \theta_+ = \Lambda(0, y, z)$, $\Lambda(0, y, z)$ generates horizon deformation after the matter absorption.

From eqs. (14), (15), and (16), they are determined as

$$T(0, y, z) = -\frac{\kappa}{4} \int_0^\infty \partial_- h_{++}(q, 0, y, z) dq, \tag{18}$$

$$R_A(0, y, z) = - \int_0^\infty h_{+A}(q, 0, y, z) dq - \frac{1}{2} \partial_A \int_0^\infty q h_{++}(q, 0, y, z) dq, \tag{19}$$

$$\Lambda(0, y, z) = -\frac{1}{2} \int_0^\infty h_{++}(q, 0, y, z) dq, \tag{20}$$

$$\partial_s \Lambda(0, y, z) = - \int_0^\infty h_{+-}(q, 0, y, z) dq - \frac{1}{2} \int_0^\infty q \partial_- h_{++}(q, 0, y, z) dq, \tag{21}$$

where the gauge conditions of $h_{\mu\nu}$ remain arbitrary except that $h_{\mu\nu}$ vanishes outside of $[x_i^+, x_f^+]$. It is easy to check that eqs. (18), (19), (20), and (21) are invariant under a gauge transformation such that $\delta_\epsilon h_{\mu\nu} = \partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu$, with $\epsilon_\mu = 0$ for $x^+ \notin [x_i^+, x_f^+]$. Here $T(0, y, z)$ generates a superrotation charge, and $R_A(0, y, z)$ generates a supertranslation charge, as seen in the next subsection. Λ corresponds to a gauge freedom from the viewpoint of the entire Minkowski spacetime and is associated with a generalized spatial translation of a Rindler region. However, it turns out that the corresponding charge (generator) is not well defined in the canonical formulation. Thus,

Λ should be regarded as just one of the dynamical variables, which controls horizon deformation, and is not associated with any asymptotic symmetry in this formulation.

2.2 Canonical Theory of Holographic Charge

Let us consider the Regge–Teitelboim canonical theory in the conformal Rindler coordinates $(\sigma^\mu) = (\tau, \rho, y, z)$. The ADM decomposition of the metric is given by

$$ds^2 = -N^2 d\tau^2 + h_{ab}(d\sigma^a + N^a d\tau)(d\sigma^b + N^b d\tau),$$

where the lowercase Roman letters run over ρ, y and z . The conjugate momentum of h_{ab} is defined as

$$\Pi^{ab} = \frac{\sqrt{h}}{16\pi G} [Kh^{ab} - K^{ab}],$$

where K_{ab} is the extrinsic curvature,

$$K_{ab} = \frac{1}{2N} (N_{a|b} + N_{b|a} - \partial_\tau h_{ab}),$$

where $|$ denotes the three-dimensional covariant derivative using h_{ab} . The Hamiltonian density \mathcal{H} and momentum density \mathcal{H}_a are defined as

$$\begin{aligned} \mathcal{H} &= \frac{\kappa^2}{\sqrt{h}} \left[\Pi^{ab} \Pi_{ab} - \frac{1}{2} \Pi^2 \right] - \frac{\sqrt{h}}{\kappa^2} R^{(3)}, \\ \mathcal{H}_a &= -2\Pi_{ab}|^b, \end{aligned}$$

and the Einstein equation in pure gravity imposes the following constraints:

$$\mathcal{H} \approx 0, \mathcal{H}_a \approx 0.$$

In order to analyze generators of coordinate transformation in the canonical theory, let us consider an infinitesimal transformation $\delta_\xi \sigma^\mu = \xi^\mu(\tau, \rho, y, z)$ and introduce

$$\begin{aligned}
\hat{\xi}^\tau &= N\xi^\tau \\
\hat{\xi}_a &= \xi_a = g_{a\mu}\xi^\mu \\
\hat{\xi}^a &= h^{ab}\hat{\xi}_b = \xi^a + N^a\xi^\tau.
\end{aligned}$$

The generator $G[\xi]$ for the transformation $\delta_\xi\sigma^\mu$ is the sum of the bulk term $H[\xi]$ and surface term $Q[\xi]$:

$$G[\xi] = H[\xi] + Q[\xi].$$

The bulk term is given by

$$H[\xi] = \int d^3\sigma \left[\hat{\xi}^\tau \mathcal{H} + \hat{\xi}^a \mathcal{H}_a \right]$$

and becomes zero if the equation of motion is satisfied. The surface term is obtained by integration of

$$\begin{aligned}
\delta Q[\xi] &= \int d^3\sigma \partial_c \left(G^{abcd} \left[\hat{\xi}^\tau \delta h_{ab|d} - \hat{\xi}^\tau|_d \delta h_{ab} \right] \right) \\
&+ \int d^3\sigma \partial_c \left(2\hat{\xi}_a \delta \Pi^{ac} \right) \\
&+ \int d^3\sigma \partial_c \left(\left[\hat{\xi}^a \Pi^{bc} + \hat{\xi}^b \Pi^{ac} - \hat{\xi}^c \Pi^{ab} \right] \delta h_{ab} \right), \tag{22}
\end{aligned}$$

where

$$G^{abcd} = \frac{1}{2\kappa^2} \sqrt{h} (h^{ac} h^{bd} + h^{ad} h^{bc} - 2h^{ab} h^{cd}).$$

In general, the integrability of eq. (22) is nontrivial, and integration is possible only for limited transformations. Near an event horizon of a Schwarzschild black hole, the integrability is proven for supertranslation and superrotation [4]. Similarly, a Rindler spacetime admits the integrability of the following asymptotic transformation on a horizon with $\rho = \infty$. The coordinate transformation of the asymptotic symmetry is given by

$$\tau' = \tau + T(y, z), \tag{23}$$

$$\rho' = \rho, \tag{24}$$

$$x'_A = X_A(y, z), \tag{25}$$

where $T(y, z)$ is an arbitrary function of y, z and generates supertranslation on the future Rindler horizon at $x^- = 0$ and on the past Rindler horizon at $x^+ = 0$. $X_A(y, z)$ generates a general coordinate transformation in the (y, z) plane and corresponds to superrotation on the same horizon. Under this transformation, the Rindler metric in eq. (4) is transformed into a stationary asymptotic metric given by

$$\begin{bmatrix} g_{\tau\tau} & g_{\tau\rho} & g_{\tau y} & g_{\tau z} \\ g_{\rho\tau} & g_{\rho\rho} & g_{\rho y} & g_{\rho z} \\ g_{y\tau} & g_{y\rho} & g_{yy} & g_{yz} \\ g_{z\tau} & g_{z\rho} & g_{zy} & g_{zz} \end{bmatrix} = \begin{bmatrix} -\Delta + O(\Delta^2) & O(\Delta^2) & O(\Delta) & O(\Delta) \\ O(\Delta^2) & \Delta & O(\Delta^2) & O(\Delta^2) \\ O(\Delta) & O(\Delta^2) & O(\Delta^0) & O(\Delta^0) \\ O(\Delta) & O(\Delta^2) & O(\Delta^0) & O(\Delta^0) \end{bmatrix}, \quad (26)$$

where $\Delta = \exp(-\frac{\rho}{\kappa})$. The supertranslation charge is defined by eq. (22) with

$$\xi^\tau = \xi^\tau(y, z), \xi^\rho = 0, \xi^A = 0.$$

The superrotation charge corresponds to

$$\xi^\tau = 0, \xi^\rho = 0, \xi^A = \xi^A(y, z).$$

By use of $\partial_\rho \sqrt{\det[h_{AB}]}(y, z) = 0$ and the above asymptotic form in eq. (26), δQ in eq. (22) is computed as

$$\delta Q = \int dydz \left[-\delta \left(\frac{\xi^\tau(y, z)}{\kappa^3} \sqrt{\det[h_{AB}]} \right) + 2\xi^A(y, z) \delta \Pi_A^\rho \right]_{\rho=\infty} \quad (27)$$

and can be integrated as

$$Q[\xi] = \int dydz \left[-\frac{\xi^\tau(y, z)}{\kappa^3} \left(\sqrt{\det[h_{AB}]} - 1 \right) + 2\xi^A(y, z) \Pi_A^\rho \right].$$

By using eqs. (23) and (25), the charges are evaluated as

$$Q[\xi] = -\frac{1}{\kappa^3} \int dydz \left[\begin{array}{c} \xi^\tau(y, z) \left(\sqrt{\det[\partial_A X^C(y, z) \partial_B X^C(y, z)]} - 1 \right) \\ + \xi^A(y, z) \partial_A T(y, z) \sqrt{\det[\partial_B X^D(y, z) \partial_C X^D(y, z)]} \end{array} \right]. \quad (28)$$

Note that the transformation in eqs. (23), (24), and (25) is fixed only near the horizon. Actually, we can extend it into the bulk region outside of the horizon and give an arbitrary (τ, ρ) dependence to ξ^μ . By using this gauge freedom, it is possible to assume without loss of generality that

$$\lim_{\rho \rightarrow -\infty} \xi^\mu(\tau, \rho, y, z) = 0.$$

Therefore, the holographic symmetry on the horizon is independent of the asymptotic symmetry at spatial infinity ($\rho \rightarrow -\infty$). The metrics generated by the symmetry on the horizon give the same asymptotic gravitational field at spatial infinity. Returning to the case of a black hole with finite mass, this means that the black hole has infinite degeneracy near the horizon with the same ADM energy and angular momentum as at spatial infinity. This degeneracy is so large that state counting may give the same order of the entropy as $\mathcal{A}/(4G)$ [5], as will be seen again in the next subsection.

If we consider an asymptotic metric with $\partial_\rho \sqrt{\det[h_{AB}]} \neq 0$, which includes the effects of incoming matter and a gravitational wave across a Rindler horizon at $x^- = 0$, δQ has an additional ρ -derivative term of δh_{AA} , and the nonperturbative integrability is broken for supertranslation. In this case, the first term on the right-hand side of eq. (22) is computed as

$$\begin{aligned} & \int d^3\sigma \partial_c \left(G^{ab\rho d} \left[\hat{\xi}^\tau \delta h_{ab|d} - \hat{\xi}^\tau|_d \delta h_{ab} \right] \right) \\ &= - \int d^3\sigma \partial_c \delta \left[\frac{\xi^\tau}{\kappa^2} \sqrt{\det[h_{AB}]} \right] \\ &+ \int d^3\sigma \partial_c \left(\xi^\tau \frac{\sqrt{\det[h_{AB}]}}{2\kappa} \left[(\partial_\rho h^{AB}) \delta h_{AB} - 2\partial_\rho (h^{AB} \delta h_{AB}) \right] \right). \end{aligned} \quad (29)$$

The first term on the right-hand side of eq. (29) is integrable and has already appeared in eq. (27). The second term is new and not always integrable for supertranslation. However, if we concentrate on the weak gravity field $h_{\mu\nu}^{(R)}$ in the Rindler gauge, integration of δQ remains achievable. The charge on the horizon is computed as

$$\begin{aligned} Q[\xi] &= \frac{1}{2\kappa^2} \lim_{x^- \rightarrow 0} \int dy dz \xi^\tau [x^+ \partial_+ + x^- \partial_- - 1] h_{AA}^{(R)}(x^+, x^-, y, z) \\ &- \frac{2}{\kappa x^+} \lim_{x^- \rightarrow 0} \int dy dz \xi^A [x^+ \partial_+ + x^- \partial_- + 1] h_{-A}^{(R)}(x^+, x^-, y, z), \end{aligned} \quad (30)$$

where $h_{\mu\nu}^{(R)} = h_{\mu\nu} + \partial_\mu \theta_\nu + \partial_\nu \theta_\mu$. It is possible to obtain the same result using the Wald–Zoupas covariant current formulation in [28]. In the region with $x^+ > x_f^+$ with some positive x_f^+ , $h_{\mu\nu}$ vanishes. Thus, $h_{\mu\nu}^{(R)}$ is given simply by $\partial_\mu \theta_\nu + \partial_\nu \theta_\mu$, where θ_μ are given by eqs. (14), (15), and (16). The charge is calculated as

$$Q[\xi] = -\frac{1}{\kappa^2} \int dydz \xi^\tau \partial_A R_A(0, y, z) + \frac{4}{\kappa^2} \int dydz \xi^A \partial_A T(0, y, z), \quad (31)$$

where $T(0, y, z)$ is given by eq. (18), and $R_A(0, y, z)$ is given by eq. (19). This result establishes the time independence of the holographic charges in the future region. If the vector field $\xi^A(y, z)$ tends to zero at spatial infinity in the (y, z) plane, it can be decomposed into

$$\xi^A(y, z) = \epsilon^{AB} \partial_B \xi^{(1)}(y, z) + \partial^A \xi^{(2)}(y, z).$$

Using this decomposition, we obtain the following expression for the superrotation charge:

$$Q_{sr}[\xi] = \frac{1}{\kappa} \int dydz [\partial_A \partial_A \xi^{(2)}(y, z)] \left[\int_0^\infty \partial_- h_{++}(q, 0, y, z) dq \right]. \quad (32)$$

From this result, the superrotation charge is found to vanish for the area-preserving component generated by $\xi^{(1)}(y, z)$. Only the rotationless component generated by $\xi^{(2)}(y, z)$ can take nonzero values of the superrotation charge.

So far, we have not yet used the Einstein equation,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{\kappa^2}{2} T_{\mu\nu}.$$

Substituting eq. (3) into the above equation and taking the first-order terms in terms of κ yields the following equation of motion for $h_{\mu\nu}$:

$$\partial^2 h_{\mu\nu} - \partial_\mu (\partial^\alpha h_{\alpha\nu}) - \partial_\nu (\partial^\alpha h_{\alpha\mu}) + \eta_{\mu\nu} \partial^\alpha \partial^\beta h_{\alpha\beta} + \partial_\mu \partial_\nu h_\alpha^\alpha - \eta_{\mu\nu} \partial^2 h_\alpha^\alpha = -\kappa T_{\mu\nu}.$$

From this equation, the energy momentum conservation of matter, $\partial^\mu T_{\mu\nu} = 0$, is automatically satisfied. If the standard harmonic gauge for $h_{\mu\nu}$,

$$\partial^\mu h_{\mu\nu} - \frac{1}{2} \partial_\nu h_\mu^\mu = 0, \quad (33)$$

is adopted, the equation of motion reads

$$\partial^\alpha \partial_\alpha h_{\mu\nu} = -\kappa \left(T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T_\lambda^\lambda \right).$$

For example, in the coordinates (x^+, x^-, y, z) , h_{++} obeys

$$[4\partial_+ \partial_- + \partial_A \partial_A] h_{++} = -\kappa T_{++}, \quad (34)$$

and h_{+-} obeys

$$[4\partial_+ \partial_- + \partial_A \partial_A] h_{+-} = \frac{\kappa}{4} T_{AA}, \quad (35)$$

where the index A takes y and z . From eq. (34), we obtain

$$\partial_A \partial_A \int_0^\infty q h_{++}(q, 0, y, z) dq = \int_0^\infty [4\partial_- h_{++}(q, 0, y, z) - \kappa q T_{++}(q, 0, y, z)] dq. \quad (36)$$

By using the gauge condition in eq. (33), the following relation holds.

$$4\partial_- h_{++}(q, 0, y, z) = \partial_+ h_{AA}(q, 0, y, z) - 2\partial_A h_{A+}(q, 0, y, z).$$

Substituting the above equation into eq. (36) gives

$$\int_0^\infty \partial_A h_{A+}(q, 0, y, z) dq + \frac{1}{2} \partial_A \partial_A \int_0^\infty q h_{++}(q, 0, y, z) dq = -\frac{\kappa}{2} \int_0^\infty q T_{++}(q, 0, y, z) dq.$$

Therefore, the supertranslation charge is computed as

$$Q_{st}[\xi^\tau] = -\frac{1}{2\kappa} \int dy dz \xi^\tau(y, z) \left[\int_0^\infty q T_{++}(q, 0, y, z) dq \right].$$

To evaluate the superrotation charge, we adopt the Green function that obeys

$$[4\partial_+ \partial_- + \partial_A \partial_A] G(x^+, x^-, y, z) = \delta(x^+) \delta(x^-) \delta^2(x^A).$$

This Green function takes the Fourier form

$$G(x^+, x^-, y, z) = -\frac{1}{(2\pi)^4} \int \frac{\exp[i(k_+ x^+ + k_- x^-)]}{4k_+ k_- + k_A k_A} e^{i(k_y y + k_z z)} dk_+ dk_- dk_y dk_z.$$

Using this expression, we obtain a convenient formula:

$$\int_{-\infty}^{\infty} dq \partial_- G(q - x'^+, 0 - x'^-, y - y', z - z') = \partial_{x'^-} \delta(x'^-) G^{(2)}(y - y', z - z'), \quad (37)$$

where $G^{(2)}$ is a two-dimensional Green function in the (y, z) plane satisfying

$$\partial_A \partial_A G^{(2)}(y - y', z - z') = \delta(y - y') \delta(z - z')$$

and given by

$$G^{(2)}(y - y', z - z') = \int \frac{e^{i[k_y(y-y') + k_z(z-z')]} dk_y dk_z}{k_y^2 + k_z^2} \frac{1}{(2\pi)^2} = \frac{1}{4\pi} \ln \left[\frac{(y - y')^2 + (z - z')^2}{\kappa^2} \right].$$

Using eq. (37), we obtain

$$\begin{aligned} & \int_{-\infty}^{\infty} \partial_- h_{++}(q, 0, y, z) dq \\ &= -\frac{\kappa}{4\pi} \int \ln \left[\frac{(y - y')^2 + (z - z')^2}{\kappa^2} \right] \left[\int_{-\infty}^{\infty} \partial_- T_{++}(q, 0, y', z') dq \right] dy' dz'. \end{aligned}$$

On the basis of these results, we obtain a general formula for the charges.

$$\begin{aligned} & Q[\xi] \\ &= -\frac{1}{2\kappa} \int dy dz \xi^\tau(y, z) \left[\int_0^\infty x^+ T_{++}(x^+, 0, y, z) dx^+ \right] \\ &+ \frac{1}{4\pi} \int dy dz \int dy' dz' \xi^A(y, z) \partial_A \\ &\times \ln \left[\frac{(y - y')^2 + (z - z')^2}{\kappa^2} \right] \left[\int_0^\infty \partial_- T_{++}(x^+, 0, y', z') dx^+ \right]. \quad (38) \end{aligned}$$

This is the main result of this paper. This expression is invariant under changes in the cutoff $\kappa \rightarrow \kappa'$ in the two-dimensional Green function.

First, it should be stressed that in eq. (38), the holographic charges of both supertranslation and superrotation vanish if we have no matter energy-momentum tensor. As seen in eq. (36), supertranslation charges of incoming gravitational wave vanish due to the Einstein equation. It is also verified that the superrotation charges vanish by considering an incoming gravitational wave field as

$$\begin{aligned} & h_{++}^{(in)}(x^+, x^-, y, z) \\ &= \sum_s \int_{-\infty}^0 dk_x \int_{-\infty}^{\infty} dk_y \int_{-\infty}^{\infty} dk_z \\ & \times \left[\varepsilon_{++}(s) a_{\vec{k},s} \exp \left[i \left(k_x x + k_y y + k_z z - \sqrt{k_x^2 + k_y^2 + k_z^2} t \right) \right] + h.c. \right], \quad (39) \end{aligned}$$

where the upper end of k_x integration is zero and describes the incoming wave condition. When this is substituted into eq. (32), using the fact that $h_{++}(x^+, x^-, y, z)$ support is only in the region $x^+ > 0$, it is noticed that

$$\int_{-\infty}^{\infty} \partial_- h_{++}(q, 0, y, z) dq \propto \int_{-\infty}^0 dk_x k_x \delta \left(k_x - \sqrt{k_x^2 + k_y^2 + k_z^2} \right) = 0,$$

so $Q_{sr}[\xi]$ certainly vanishes for gravitational waves. This means that the charges do not store gravitational wave information, at least to the first order of κ . This result for gravitational waves is similar to the result for electromagnetic waves. On a Rindler horizon, the HPS charge in [2] is given by

$$Q_{HPS}(x^+) = \int dy dz \xi(y, z) E^x(x^+, 0, y, z),$$

where E^x is the x component of the electric field on the horizon at $x^- = 0$, and $\xi(y, z)$ is a $U(1)$ gauge parameter depending on (y, z) . An electromagnetic wave can yield nontrivial time evolution of Q_{HPS} during its passage across the horizon. However, in the late-time region with $x^+ > x_f^+$, the wave is already located inside of the horizon, and no electric field is on the horizon. Therefore, the charge tends to zero, and no information about the electromagnetic wave is stored in holographic states on the horizon¹. The reason

¹If we consider a S^2 boundary with infinite radius at null future infinity enclosing interior electric charges, electromagnetic waves crossing the boundary can shift the values of the HPS charges on the boundary.

that the electromagnetic holographic charge vanishes for radiation is essentially that the charge arises from a gauge-invariant electric field. However, it should be stressed that the situation for gravitational waves may differ from that for electromagnetic waves. In general relativity, gauge-dependent variables such as the metric are also physical observables. By using many clocks that are distributed in space and exchanging signals between them, the metric is determined experimentally. As shown in eq. (30), the holographic charges are not determined by the curvature tensors $R_{\alpha\beta\mu\nu}$. They consist of the metric and its first derivatives. Thus, the charges are gauge-dependent objects, although they can be observed by physical detectors. Further, a stationary asymptotic Rindler spacetime without any matter in it has nontrivial charges of supertranslation and superrotation [4]. Hence, even in pure gravity, it is possible that a gravitational wave generates a shift in the values of the charges in the future region. To investigate this possibility, a long calculation is required. After the computation is complete, we find, as a nontrivial result, that gravitational waves never shifts the charges of supertranslation and superrotation.

Note that the gravitational holographic states store the information about the electromagnetic waves, as seen in eq. (38) with

$$T_{\mu\nu} = F_{\mu}^{\alpha} F_{\nu\alpha} - \frac{1}{4} \eta_{\mu\nu} F^{\alpha\beta} F_{\alpha\beta},$$

where $F_{\alpha\beta}$ is a tensor of electric field and magnetic field. This may tempt us to imagine that the same scenario applies for gravitational waves by taking account of higher-order correction terms in terms of κ , which induces a pseudo energy-momentum tensor for gravitational waves. Unfortunately, we encounter serious trouble. The integrability of the supertranslation charge in eq. (29) breaks down because of the higher-order corrections in the second term. Thus, it remains a crucial open question whether holographic charges on a horizon can be defined for strong gravitational waves. This will be discussed again in section 4.

Four more comments regarding the main result in eq. (38) are listed below. (i) The classical energy condition ensures positivity of the generator of the Lorentz boost (Rindler energy) on the horizon in the supertranslation charge:

$$E_R = \int_0^\infty x^+ T_{++}(x^+, 0, y, z) dx^+ \geq 0.$$

Thus, the bulk Rindler energy associated with $\xi^\tau(y, z) = \text{const}$ in eq. (38)

always decreases when incoming matter crosses the horizon. (ii) The superrotation charge in eq. (38) for matter propagating in parallel in the x direction, which obeys $\partial_- T_{++}(x^+) = 0$, is equal to zero. In the superrotation charged states, mainly interference information about waves propagating in different directions is stored. (iii) The local supertranslation charge $\delta Q[\xi]/\delta \xi^\tau(y, z)$ retains only the information about the total amount of Rindler energy E_R that passes through the point (y, z) . However, the superrotation charge stores information about matter nonlocally. The influence of $\int_0^\infty \partial_- T_{++} dx^+$ at some point propagates widely in the (y, z) plane via the logarithmic long-range behavior of the two-dimensional Green function $G^{(2)}$. This resembles the behavior of the black hole S matrix of 't Hooft [29]. (iv) In a Minkowski background, we have an infinite number of Rindler horizons. Thus, we can define a measure of gravitational memory at different horizons using the supertranslation and superrotation charges. Let us define $M(x_h^+, x_h^-)$ for a future Rindler horizon at $x^- = x_h^-$ and a Rindler wedge at $(x^+, x^-) = (x_h^+, x_h^-)$ as

$$\begin{aligned}
M(x_h^+, x_h^-) &= -\frac{1}{2\kappa} \int dy dz \xi^\tau(y, z) \left[\int_0^\infty (x^+ + x_h^+) T_{++}(x^+ + x_h^+, x_h^-, y, z) dx^+ \right] \\
&+ \frac{1}{4\pi} \int dy dz \int dy' dz' \xi^A(y, z) \partial_A \\
&\times \ln \left[\frac{(y - y')^2 + (z - z')^2}{\kappa^2} \right] \left[\int_0^\infty \partial_- T_{++}(x^+ + x_h^+, x_h^-, y', z') dx^+ \right]. \quad (40)
\end{aligned}$$

Using this memory, each horizon acts as a holographic screen that stores matter information, as depicted in figure 4. In the next section, $M(x_h^+, x_h^-)$ is quantized, and the no-cloning paradox is discussed.

2.3 Thick Black Hole Hair on Horizon

Here let us apply the state counting argument in [5] to stationary asymptotic Rindler spacetimes. This counting is just a rough estimation. However, it strongly suggests that the horizon states can supply an entropy of $O(\mathcal{A}/4G)$

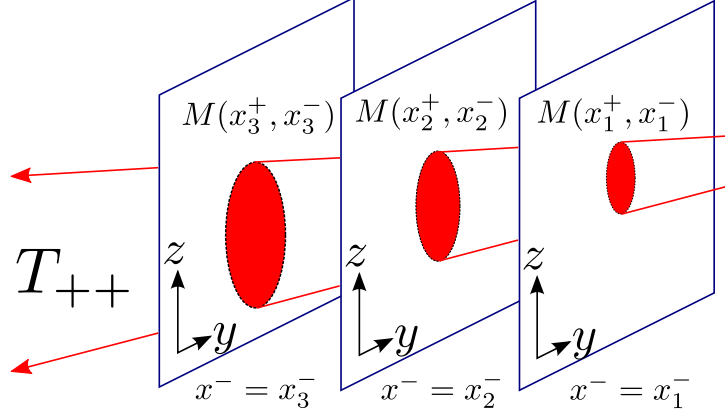


Figure 4: Gravitational memory defined with supertranslation and superrotation charges; each horizon plays the role of a holographic screen which stores matter information.

and implies that superrotation and supertranslation on a horizon may make it possible to create black holes with thick hair. Let us consider the following generators of supertranslation and superrotation on a Rindler horizon in the (τ, ρ, y, z) coordinates:

$$\begin{aligned} G_{st} [\xi^\tau] &= \xi^\tau (y, z) \partial_\tau, \\ G_{sr} [\xi^A] &= \xi^A (y, z) \partial_A. \end{aligned}$$

They form a closed algebra such that

$$[G_{st} [\xi^\tau], G_{st} [\xi'^\tau]] = 0, \quad (41)$$

$$[G_{sr} [\xi^A], G_{st} [\xi^\tau]] = G_{st} [\xi^A \partial_A \xi^\tau], \quad (42)$$

$$[G_{sr} [\xi^A], G_{sr} [\xi'^A]] = G_{sr} [\xi^B \partial_B \xi'^A - \xi'^B \partial_B \xi^A]. \quad (43)$$

Assume that the (y, z) plane has a finite area $\mathcal{A} = L_y L_z$ by imposing periodic boundary conditions in the y and z directions. Then the Fourier expansion of ξ^τ is given by

$$\xi^\tau(y, z) = \sum_{n_y = -N_y/2}^{N_y/2} \sum_{n_z = -N_z/2}^{N_z/2} \frac{\xi_{n_y n_z}^\tau}{L_y L_z} \exp\left(2\pi i n_y \frac{y}{L_y}\right) \exp\left(2\pi i n_z \frac{z}{L_z}\right).$$

To regularize the ultraviolet divergence, the cutoffs N_y and N_z are introduced, and the corresponding momenta are on the order of the Planck scale. This yields the following relation.

$$N_A = \frac{L_A}{\kappa}. \quad (44)$$

The generators of each Fourier component are defined as

$$G_{st}(n_y, n_z) = \frac{1}{L_y L_z} \exp\left(2\pi i n_y \frac{y}{L_y}\right) \exp\left(2\pi i n_z \frac{z}{L_z}\right) \partial_\tau$$

and satisfy

$$[G_{st}(n_y, n_z), G_{st}(n'_y, n'_z)] = 0.$$

The algebraic structure in eqs. (41), (42), and (43) is very similar to that of Poincaré algebra, although we have an infinite number of generators on the horizon. $G_{st}[\xi^\tau]$ corresponds to the momentum, and $G_{sr}[\xi'^A]$ corresponds to the angular momentum and Lorentz boost. To analyze the irreducible representation of the algebra in eqs. (41), (42), and (43), Wigner's little group technique [30] may play a crucial role, as it does in the Poincaré group case. The simplest nontrivial irreducible unitary representation derived from little group analysis is given by a Hilbert space spanned by simultaneous eigenstates of all the supertranslation charges $\hat{Q}_{st}(n_y, n_z)$, which are Hermitian operators associated with $G_{st}(n_y, n_z)$ and obey

$$[\hat{Q}_{st}(n_y, n_z), \hat{Q}_{st}(n'_y, n'_z)] = 0. \quad (45)$$

The eigenvalues of $\hat{Q}_{st}(n_y, n_z)$ are identified as classical values of $Q_{st}(n_y, n_z)$, which is obtained by rescaling the supertranslation charge in eq. (28) by the black hole scale \mathcal{A} as

$$Q_{st}(n_y, n_z) = \int \frac{dydz}{L_y L_z} \sqrt{\det[\partial_A X^C(y, z) \partial_B X^C(y, z)]} \exp\left(2\pi i n_y \frac{y}{L_y}\right) \exp\left(2\pi i n_z \frac{z}{L_z}\right),$$

where $(Y, Z) = (X^y(y, z), X^z(y, z))$ is a regular coordinate transformation in the (y, z) plane. Under the above hypothesis, we can evaluate $|Q_{st}(n_y, n_z)|$ as

$$\begin{aligned}
|Q_{st}(n_y, n_z)| &= \left| \int \frac{dydz}{L_y L_z} \sqrt{\det [\partial_A X^C(y, z) \partial_B X^C(y, z)]} \exp \left(2\pi i n_y \frac{y}{L_y} \right) \exp \left(2\pi i n_z \frac{z}{L_z} \right) \right| \\
&= \left| \int \frac{dY dZ}{L_y L_z} \exp \left(2\pi i n_y \frac{y(Y, Z)}{L_y} \right) \exp \left(2\pi i n_z \frac{z(Y, Z)}{L_z} \right) \right| \\
&\leq \left| \int \frac{dY dZ}{L_y L_z} \right| = 1.
\end{aligned}$$

Hence, the range of eigenvalues is independent of \mathcal{A} . In quantum gravity, let us imagine that the number of quantum analogs of $(X^y(y, z), X^z(y, z))$ is such that the number of eigenvalues becomes finite and of order one with respect to \mathcal{A} :

$$\#Q_{st}(n_y, n_z) = O(\mathcal{A}^0) = O(1).$$

Then, using the cutoff in eq. (44), the degeneracy of the supertranslation charge is estimated as

$$\# \{(\cdots, Q_{st}(0, 0), Q_{st}(1, 0), Q_{st}(0, 1), \cdots)\} = O(1)^{N_y N_z} = O(1)^{\mathcal{A}/\kappa^2}$$

and yields an entropy of the same order as the black hole entropy.

$$S = \ln(\# \{(\cdots, Q_{st}(0, 0), Q_{st}(1, 0), Q_{st}(0, 1), \cdots)\}) = O\left(\frac{\mathcal{A}}{4G}\right).$$

Therefore, it is possible that a huge number of charged states on the horizon accounts for the statistical mechanical origin of $\mathcal{A}/(4G)$. Even though exact state counting of the charges on the horizon in nonperturbative quantum gravity has not been achieved yet and is a crucial open question, we no longer need to believe that black holes are hairless.

3 Quantum Memory Operators on Horizons

In this section, we discuss the no-cloning problem of Rindler horizons. Let us consider quantization of the weak gravity field. Its quantum field is given

by

$$\hat{h}_{\mu\nu} = \hat{h}_{\mu\nu}^{(GW)} + \hat{h}_{\mu\nu}^{(M)}.$$

The graviton component, $\hat{h}_{\mu\nu}^{(GW)}$, is a canonically quantized weak gravitational wave and obeys $\partial^2 \hat{h}_{\mu\nu}^{(GW)} = 0$. The other component, $\hat{h}_{\mu\nu}^{(M)}$, comes from quantized matter sources and is given by

$$\hat{h}_{\mu\nu}^{(M)} = -\kappa \int G(x - x') \left(\hat{T}_{\mu\nu}(x') - \frac{1}{2} \eta_{\mu\nu} \hat{T}_\lambda^\lambda(x') \right),$$

where \hat{T}_{++} satisfies $\langle 0 | \hat{T}_{++} | 0 \rangle = 0$ for the vacuum state $|0\rangle$ in the standard canonical quantization. Both components can contribute to supertranslation and superrotation quantum charges defined as

$$\begin{aligned} \hat{Q}[\xi] &= \frac{1}{2\kappa^2} \int dydz \xi^\tau(y, z) [x^+ \partial_+ - 1] \hat{h}_{AA}^{(R)}(x^+, 0, y, z) \\ &\quad - \frac{2}{\kappa x^+} \int dydz \xi^A(y, z) [x^+ \partial_+ + 1] \hat{h}_{-A}^{(R)}(x^+, 0, y, z), \end{aligned} \quad (46)$$

where $\hat{h}_{\mu\nu}^{(R)}$ in the Rindler gauge is defined from $\hat{h}_{\mu\nu}$ using eqs. (14), (15), and (16) as in classical theory. In this paper, we concentrate on the memory effect of $\hat{h}_{\mu\nu}^{(M)}$. The quantum effect of $\hat{h}_{\mu A}^{(GW)}$ on the holographic charges will be reported elsewhere. Let us define the quantum gravitational memory operators on a future horizon at $x^- = x_h^-$ with a Rindler wedge located at $(x^+, x^-) = (x_h^+, x_h^-)$ as

$$\begin{aligned} \hat{M}(x_h^+, x_h^-) &= -\frac{1}{2\kappa} \int dydz \xi^\tau(y, z) \left[\int_0^\infty (x^+ + x_h^+) \hat{T}_{++}(x^+ + x_h^+, x_h^-, y, z) dx^+ \right] \\ &\quad + \frac{1}{4\pi} \int dydz \int dy' dz' \xi^A(y, z) \partial_A \\ &\quad \times \ln \left[\frac{(y - y')^2 + (z - z')^2}{\kappa^2} \right] \left[\int_0^\infty \partial_- \hat{T}_{++}(x^+ + x_h^+, x_h^-, y', z') dx^+ \right]. \end{aligned} \quad (47)$$

Consider a massless matter field $\hat{\varphi}$ that is initially in the vacuum state $|0\rangle$. For the field $\hat{\varphi}$, let us consider a local unitary operator \hat{U} that includes

some information to be measured by observers. Applying \hat{U} to $|0\rangle$ generates a quantum wavepacket in an excited state, $|\Psi\rangle = \hat{U}|0\rangle$. The wavepacket crosses a future Rindler horizon, as depicted in figure 5. From the viewpoint of the entire Minkowski spacetime, the information about \hat{U} is continuously carried by the wavepacket. However, from the viewpoint of the Rindler spacetime, we have holographic charges on the horizon that store, at least partially, the information about \hat{U} , as depicted in figure 6. Then, an important problem is how much information about \hat{U} is stored on the horizon. In the black hole complementarity scenario [15] of nonperturbative quantum gravity, all of the information can be copied and stored on the horizon. Although this sounds incompatible with the no-cloning theorem [16], it may be possible to reconcile this discrepancy for realistic black hole cases because there is a singularity inside of the horizon that might completely delete quantum information that was carried by matter colliding with the singularity [15]. However, in the Minkowski spacetime, no singularity exists. Thus, the black hole complementarity approach does not succeed. Further, we have an infinite number of Rindler horizons in the Minkowski case. In principle, the information about \hat{U} can be simultaneously shared by many horizons, as depicted in figure 7. Of course, the classical component of the information about \hat{U} is replicable and easy to share. What happens if the holographic charges of two or more horizons store all of the purely quantum information about \hat{U} ? This looks very troublesome at a deep level of the full quantum gravity theory, and the no-cloning paradox may be inevitable if the holographic charges have reality for an observer who passes the horizons and observes the holographic charges for each horizon. To avoid that, we propose a simple conjecture about this question from the viewpoint of quantum measurement contextuality. First, it is worth noting that the energy-momentum tensor of quantum matter inside of a horizon does not commute with $\hat{M}(x_h^+, x_h^-)$. This also leads to noncommutativity between the holographic charges of different horizons. For instance, let us consider a free massless scalar field $\hat{\varphi}$. Its $(++)$ component of the energy-momentum tensor is given by

$$\hat{T}_{++}(x) =: \partial_+ \hat{\varphi}(x) \partial_+ \hat{\varphi}(x) :$$

, where $: \quad :$ stands for normal ordering of the operators. A commutator for $\hat{T}_{++}(x)$ and $\hat{T}_{++}(y)$ is computed as

$$\left[\hat{T}_{++}(x), \hat{T}_{++}(y)\right] = \langle 0 | [\partial_+ \hat{\varphi}(x), \partial_+ \hat{\varphi}(y)] | 0 \rangle (\partial_+ \hat{\varphi}(x) \partial_+ \hat{\varphi}(y) + \partial_+ \hat{\varphi}(y) \partial_+ \hat{\varphi}(x)). \quad (48)$$

Here $\langle 0 | [\partial_+ \hat{\varphi}(x), \partial_+ \hat{\varphi}(y)] | 0 \rangle$ is proportional to $\partial_+^2 \delta \left((x - y)^\mu (x - y)_\mu \right)$. The integration in the definition of $\hat{M}(x_h^+, x_h^-)$ in eq. (47) includes $\hat{T}_{++}(y)$, which has a nonvanishing commutation relation with $\hat{T}_{++}(x)$ in eq. (48). Hence,

$$\left[\hat{T}_{++}(x), \hat{M}(x_h^+, x_h^-)\right] \neq 0,$$

and this results in the noncommutativity of the memory operators:

$$\left[\hat{M}(x_h^+, x_h^-), \hat{M}(x_h'^+, x_h'^-)\right] \neq 0.$$

When we measure $\hat{M}(x_h^+, x_h^-)$ on a horizon, other $\hat{M}(x_h'^+, x_h'^-)$ values and quantum states of matter inside the horizon are affected because of the wavefunction collapse induced by the measurement. This suggests that the holographic charge reality is conditioned to measurements by appropriate physical detectors. As is well established, electric charge takes a universal value for a particle independently of observers and measurements. So it can be treated as reality. However, the holographic charge is not. Rather, it emerges via measurements by appropriate physical detectors for measurements of the near-horizon metric. As depicted in figure 8, the charge becomes physical only when we distribute the metric measurement devices in the space. Without measurement devices, the charge is merely a gauge freedom of the general covariance in the Minkowski background. Then no cloning paradox arises, at least in the first order of perturbative quantum gravity. This is very similar to the case of Unruh–DeWitt particle detectors [22] [23] for Unruh radiation in a Rindler spacetime. For Hawking–Unruh particles in the Minkowski vacuum state as well, quantum metric measurements result in the reality of the holographic charges.

Here we comment on the duration of holographic charge measurements. As seen in eq. (47), the charges are evaluated from time integrations of \hat{T}_{++} , which passes through the horizon. Thus, the measurement is not achieved by any instantaneous measurement of the energy-momentum tensor of infalling matter on the horizon. However, one might expect that instantaneous measurements of the charges are still possible in order to directly detect the

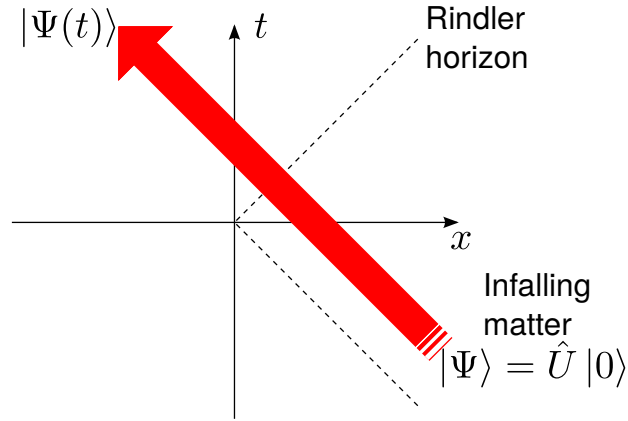


Figure 5: A quantum wavepacket in an excited state $|\Psi\rangle$, created by applying a local unitary operator \hat{U} comes across a future Rindler horizon.

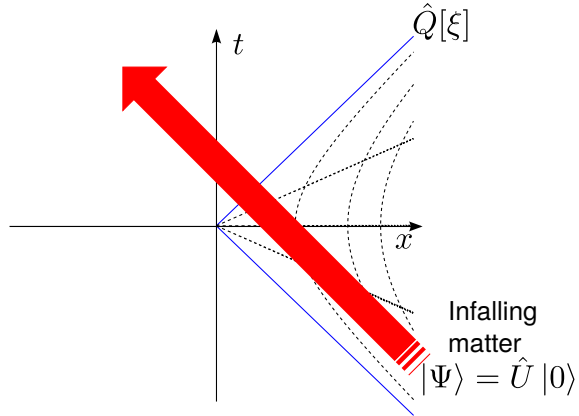


Figure 6: From Minkowski spacetime, the information of \hat{U} is carried by the wavepacket $|\Psi\rangle$. However, from Rindler spacetime we have a holographic charge on the horizon, which stores the information of \hat{U} .

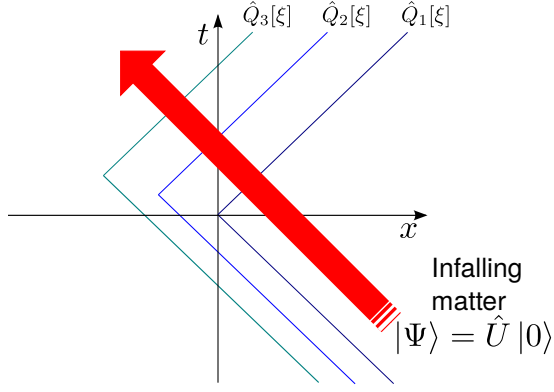


Figure 7: In principle, the information of \hat{U} can be simultaneously shared by an infinite number of Rindler horizons.

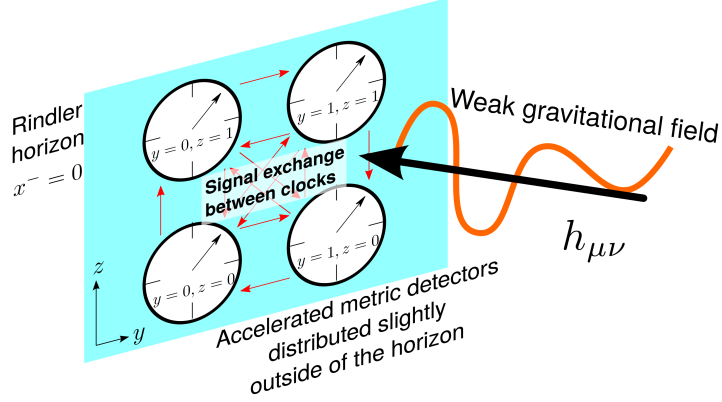


Figure 8: The holographic charge reality is conditioned to measurements. Only when we perform metric measurements by detectors near horizons does the charge become physical.

near-horizon metric in the Rindler gauge at a later time. Actually, in eq. (46), the charges at a later time can be computed from an equal-time metric on the horizon in the Rindler gauge. Thus, if the metric in the Rindler gauge at that time can be physically observed, the charges are fixed during an arbitrarily short time. However, this does not work because the metric in the Rindler gauge cannot be fixed uniquely. Ambiguity exists because there are four arbitrary functions, T' , R'_y , R'_z , and Λ' in eqs. (14), (15), and (16), so the values of the charges are not determined by instantaneous measurements. What we can do is to measure how much the charges increase during evolution of the infalling matter. A change in the charge is not observed by any instantaneous measurement at a fixed time. A measurement of the holographic charge outputs meaningful data only when the metric evolution is continuously monitored by metric detectors during the entire evolution. Again, owing to gravitational interaction, the detectors inevitably interact with infalling matter during its evolution and share quantum entanglement, as depicted in figure 9. Thus, infalling matter is decohered.

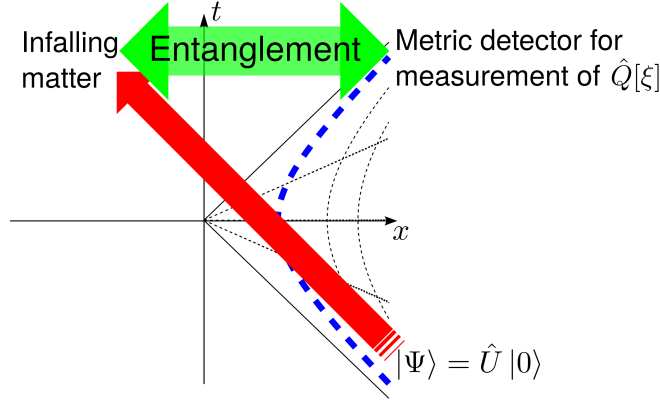


Figure 9: Measurement of holographic charge is meaningful only when metric evolution is continuously monitored. Due to gravitational interaction, the metric detectors interact with the infalling matter and share quantum entanglement.

4 Summary and Discussion

In this paper, a general theory of gravitational holographic charges for a (1+3)-dimensional linearized gravity field was formulated. The main result appears in eq. (38). As a lemma, it is found that holographic states on the horizon cannot store any information about absorbed perturbative gravitational waves. When we take into account second-order weak gravitational waves, the integrability of holographic charges in eq. (22) is broken for supertranslation. This raises the natural question of whether holographic charges are defined nonperturbatively for strong gravitational waves. A naive guess would be that it is impossible. Because asymptotic symmetry on a horizon appears owing to the background metric isometry, it is natural to expect that a large departure from the background metric no longer has any symmetry and breaks the integrability of holographic charges for no specific reason. In section 3, we proposed a conjecture to resolve the no-cloning paradox between infalling matter and holographic charges. If the reality concept of holographic charges is abandoned, no paradox occurs. The holographic charges are merely an emergent concept, and near-horizon metric measurement devices make observations as if the charges had some reality via entanglement between matter inside of the horizon and the detectors.

As a discussion, an interesting question can be posed. Do Hawking–Unruh particles in realistic gravitational collapse, which propagate toward null future infinity, act as a metric detector near the horizon? It is certainly true that the mode functions of the particles flush through a near-horizon region early in the gravitational collapse. Then do the particles store any information about events on the horizon? The possibility that the particles remember such information has been seriously discussed for a long time by many researchers, especially by ’t Hooft [29] and Page [31]. However, in the Minkowski background with a Rindler horizon, the Hawking–Unruh particles observed by Unruh–De Witt particle detectors seem to completely forget the information on the horizon. For instance, let us consider an Aichelburg–Sexl shock wave [32] passing through a Rindler horizon. Initially, the quantum fields of the Hawking–Unruh particles are in the Minkowski vacuum. After the shock wave passes, the quantum states of the fields are not excited and remain the vacuum state because the Lorentz invariance of the spacetime prohibits particle creation by the shock wave. This means that no information about the shock wave is stored in the quantum fluctuation of the fields.

Thus, the Hawking–Unruh particles observed later also remember nothing about the shock wave. Contrary to the expectation of HPS [2], the information loss problem seems to remain elusive even if we take account of the asymptotic symmetry on the horizon, although it may reveal the statistical mechanical origin of the Bekenstein–Hawking entropy.

Acknowledgments

We would like to thank Song He and W.G. Unruh for useful discussions. This research is partially supported by JSPS KAKENHI Grant Number 16K05311, the Foundational Questions Institute and Silicon Valley Community Foundation.

References

- [1] S. W. Hawking, "The Information Paradox for Black Holes," arXiv:1509.01147.
- [2] S. W. Hawking, M. J. Perry, and A. Strominger, "Soft Hair on Black Holes," Phys. Rev. Lett. **116**, 231301, (2016).
- [3] S. W. Hawking, Phys. Rev. D **14**, 2460, (1976).
- [4] M. Hotta, K. Sasaki, and T. Sasaki, Class. Quantum Grav. **18**, 1823 (2001).
- [5] M. Hotta, Phys. Rev. D **66**, 124021 (2002).
- [6] J. D. Brown and M. Henneaux, Commun. Math. Phys. **104**, 207 (1986).
- [7] A. Strominger, J. High Energy Phys. **02**, 009 (1998).
- [8] S. Carlip, Phys. Rev. D **51**, 632 (1995).
- [9] M.-I. Park and J. Ho, Phys. Rev. Lett. **83**, 5595 (1999).
- [10] S. Carlip, Phys. Rev. Lett. **83**, 5596 (1999).
- [11] M.-I. Park and J. H. Yee, Phys. Rev. D **61**, 088501 (2000).

- [12] O. Dreyer, A. Ghosh, and J. Wisniewski, *Class. Quantum Grav.* **18**, 1929 (2001).
- [13] J. Koga, *Phys. Rev. D* **64**, 124012 (2001).
- [14] W. G. Unruh, *Phys. Rev.* **D14** (4) 870, (1976).
- [15] L. Susskind, L. Thorlacius, and J. Uglum, *Phys. Rev. D* **48**, 3743 (1993);
C. R. Stephens, G. 't Hooft, and B. F. Whiting, *Class. Quant. Grav.* **11**, 621 (1994).
- [16] W. K. Wootters and W. H. Zurek, *Nature* **299**, 802, (1982).
- [17] Ya. B. Zeldovich and A. G. Polnarev, *Sov. Astron.* **18**, 17 (1974).
- [18] M. Ludvigsen, *Gen. Rel. Grav.* **21**, 1205 (1989).
- [19] D. Christodoulou, *Phys. Rev. Lett.* **67**, 1486 (1991).
- [20] L. Blanchet and T. Damour, *Phys. Rev. D* **46**, 4304 (1992).
- [21] A. Strominger and A. Zhiboedov, *JHEP* **01**, 086 (2016).
- [22] W. Unruh, *Phys. Rev. D* **14**, 870 (1976).
- [23] B. De Witt, *General Relativity: An Einstein Centenary Survey*, ed. by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, 1979), pp. 680–745.
- [24] F. Wilczek, in *Proceedings of Houston Conference on Black Holes, Membranes, Wormholes and Superstrings*, ed. S. Kalara and D. Nanopoulos (World Scientific, Singapore, 1992), p. 1.
- [25] M. Hotta, R. Schützhold, and W. G. Unruh, *Phys. Rev. D* **91**, 124060 (2015).
- [26] M. Hotta and A. Sugita, *Prog. Theor. Exp. Phys.* 123B04 (2015).
- [27] A. Almheiri, D. Marolf, J. Polchinski, and J. Sully, *J. High Energy Phys.* **02**, 062 (2013); c.f. S. L. Braunstein, arXiv:0907.1190, published as S. L. Braunstein, S. Pirandola, and K. Zyczkowski, "Better Late than Never: Information Retrieval from Black Holes," *Phys. Rev. Lett.* **110**, 101301 (2013).

- [28] R. M. Wald and A. Zoupas, Phys. Rev. D **61**, 084027 (2000).
- [29] G. 't Hooft, Int. J. Mod. Phys. A **11**, 4623 (1996).
- [30] E. P. Wigner, Ann. Math. **40**, 149 (1939).
- [31] D. N. Page, "Black Hole Information," in *Proceedings of the 5th Canadian Conference on General Relativity and Relativistic Astrophysics*, University of Waterloo, 13–15 May, 1993, ed. by R. B. Mann and R. G. McLenaghan (World Scientific, Singapore, 1994), pp. 1–41; D. N. Page, Phys. Rev. Lett. **71**, 3743 (1993).
- [32] P. C. Aichelburg and R. U. Sexl, J. Gen. Rel. Grav. **2**, 303 (1971).